# General Integral Solution of the Regenerator Transient Test Equations for Zero Longitudinal Conduction

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Since the pioneer work of Anzelius (1) Nusselt (2), Schumann (3), and Hausen (4), a variety of theoretical solutions for thermal response of an initially isothermal matrix subject to specific forms of inlet fluid temperature disturbances have been formulated. This paper provides a single general solution accommodating any inlet temperature disturbance, which is suitable for determining the heat transfer performance of certain matrix geometries near ambient temperature.

## 1 NOMENCLATURE

| | |
|---|---|
| $a$, $a_0$, $a_1$ | Numerical constants |
| $b_1$ | Numerical constant |
| $B$ | Mean solid temperature excess $(\theta_b - \theta_i)$ K |
| $B^{\#}$ | Non-dimensional ratio $B_2/G_1$ |
| $B_i$ | Biot number $= hr/k$ |
| $Cp$ | Specific heat J/kgK |
| $D$ | Non-dimensional inlet disturbance |
| $G$ | Mean fluid temperature excess $(\theta_g - \theta_i)$ K |
| $G^{\#}$ | Non-dimensional ratio $G_2/G_1$ |
| $h$ | Heat transfer coefficient, J/m$^2$ sK |
| $k$ | Numerical constant |
| $L$ | Length of matrix, m |
| $m_b$ | Mass of matrix, kg |
| $m_g$ | Mass of gas in matrix, kg |
| $\dot{M}$ | Mass flow rate of gas, kg/s |
| $Ntu$ | Number of transfer units |
| $s$ | Image of $t$ |
| $S$ | Surface area, m$^2$ |
| $t$ | Time, s |
| $t^*$ | Time constant of inlet fluid exponential disturbance |
| $u$ | Gas velocity defined as $(\dot{M}L/m_g)$, m/s |
| $W$ | Water equivalent ratio $(m_b Cp_L/m_g Cp_g)$ |
| $x$ | Distance into matrix, m |
| $\beta$ | $\tau/Ntu$ |
| $\delta(\ )$ | Delta function |
| $\theta$ | Temperature, K |
| $\sigma$ | Dummy variable |
| $\tau$ | Non-dimensional time |
| $\Upsilon^*$ | Non-dimensional time constant |
| $\omega$ | Rotational speed, 1/s |
| $\bar{\omega}$ | Non-dimensional rotational speed |

**Subscripts**

| | |
|---|---|
| $b$ | Bulk solid |
| $g$ | Gas |
| $i$ | Reference |
| $s$ | Surface |
| 1 | Inlet |
| 2 | Outlet |

Other symbols are defined where introduced.

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## 2 INTRODUCTION

The regenerator transient test technique may be used to determine heat transfer in crushed rock beds, plate–fin exchanger cores, tube banks, etc., where a single surface heat transfer coefficient is to be determined.

In its simplest form, the experimental rig used in such applications comprises a duct containing the high thermal capacity test matrix, through which gas is arranged to pass at a steady mass flow rate. Fast response platinum resistance thermometers, placed immediately before and after the test matrix, record timewise variation in inlet and outlet temperatures. Upstream of the test matrix a fast response electrical resistance heater is used to impose a known temperature disturbance on the initially isothermal air stream. Heat transfer performance of the test matrix is determined from the change in shape of outlet temperature response with respect to inlet temperature disturbance, through comparison with a mathematical model of the system.

In precise testing it is essential that experimental conditions match the mathematical model in use. Different matrices may require different mathematical models, e.g., a plate–fin exchanger core may require inclusion of a term for longitudinal conduction. The theory presented below is particularly suitable for testing tube banks in crossflow in which the longitudinal conduction term is absent (5), (6).

### Physical Assumptions

(1) Thermal and physical properties of the gas and matrix are independent of temperature (implying that the temperature change of inlet disturbance is small compared with the absolute temperature of the gas).

(2) Thermal conductivity of the matrix material is infinitely large in the direction normal to gas flow, and infinitely small in the direction parallel to the flow (implying negligible heat loss from test matrix casing, that is, testing near ambient temperature conditions, with small axial conduction path within the matrix itself).

(3) Thermal capacity of the gas in the matrix at any instant is small compared with the thermal capacity of the matrix itself (e.g., implying, air as test fluid, copper for test matrix).

(4) Surface temperatures and bulk temperatures for the solid matrix during thermal transients are the same



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(implying $Bi \to 0$, that is, solid sections thin, and/or with high thermal conductivity).

(5) Test conditions initially isothermal.

Circumstances may require departure from the above conditions, e.g., the requirement to test at much higher temperatures may introduce heat loss from the matrix surface and therefore transverse temperature gradients within the test matrix and gas. In this case the bulk temperature within the solid may have to be related to surface temperatures and longitudinal diffusion within the gas may become significant. Additional terms in the equations will then be required.

For the physical assumptions specified, a variety of mathematical attacks on the transient test technique have been published for different input disturbances **(1)**–**(32)**.

It seems useful to bring a number of these together in a single general solution capable of accepting the range of input disturbances listed in Table 1. The analysis given is for initially isothermal conditions in the absence of longitudinal conduction.

*Table 1*

| Input disturbance | Representative experimental papers | Ref. |
|---|---|---|
| Sine wave | Bell and Katz | **7** |
| Sine wave | Meek | **8** |
| Sine wave | Hart and Szomanski | **9** |
| Step | Mondt | **10** |
| Step | Pucci, Howard, and Piersall | **11** |
| Square wave | Close | **12** |
| Exponential | Smith and Coombs | **5** |
| Exponential | Liang and Yang | **13** |

## 3 THEORY

### Coupled Fluid and Solid Equations

Heat transfer to a gas flowing steadily through a porous prism is described by

*Fluid*

$$\frac{\partial \theta_g}{\partial t} + u\frac{\partial \theta_g}{\partial x} = \frac{hS}{m_g Cp_g}(\theta_s - \theta_g) = Ntu\,\frac{u}{L}(\theta_s - \theta_g) \tag{1}$$

where $u = \dot{M}L/m_g$.

*Solid*

$$\frac{\partial \theta_b}{\partial t} = \frac{hS}{m_b Cp_b}(\theta_g - \theta_s) = \frac{Ntu}{W}\frac{u}{L}(\theta_g - \theta_s) \tag{2}$$

Introducing non-dimensional scaling of length $\xi = Ntu(x/L)$ and non-dimensional scaling and modification of time $\tau = [(Ntu/W)\{(ut - x)/L\}]$. Then when $Bi \to 0$, $\theta_s \to \theta_b$, eqs. (1) and (2) become

*Fluid*

$$\frac{\partial G}{\partial \xi} = (B - G) \tag{3}$$

*Solid*

$$\frac{\partial B}{\partial \tau} = (G - B) \tag{4}$$

### Solution of Basic Equations

Taking Laplace transforms

$$L\left\{\frac{\partial G}{\partial \xi}\right\} = \frac{d\hat{G}}{d\xi}$$

$$L\left\{\frac{\partial B}{\partial \tau}\right\} = S\hat{B} - B(\xi, 0)$$

Term $B(\xi, 0)$ is initial temperature distribution in the matrix. For isothermal conditions at start of blow $B(\xi, 0) = 0$, which keeps the solution simple, see e.g., Kohlmayr **(14)**, then

*Fluid*

$$\frac{d\hat{G}}{d\xi} = \hat{B} - \hat{G} \tag{5}$$

*Solid*

$$s\hat{B} = \hat{G} - \hat{B} \tag{6}$$

Combining eqs. (5) and (6) to obtain fluid temperatures

$$\frac{d\hat{G}}{d\xi} + \left(1 - \frac{1}{s+1}\right)\hat{G} = 0$$

which has the solution

$$\hat{G} = A\exp\left\{\left(\frac{1}{s+1} - 1\right)\xi\right\}$$

where $A$ is to be determined from boundary conditions.

*At inlet*

$$\xi = 0$$

$$\hat{G}_1(0, s) = A = \hat{g}(s)$$

defined as the Laplace transform of inlet fluid temperature disturbance.

Thus

$$\hat{G} = \hat{g}(s)\exp\left\{\left(\frac{1}{s+1} - 1\right)\xi\right\} \tag{7}$$

*At outlet*

$$\xi = Ntu$$

$$\hat{G}_2(Ntu, s) = \hat{g}(s)\,.\,\exp\left\{\left(\frac{1}{s+1} - 1\right)Ntu\right\}$$

Applying inverse Laplace transforms to outlet fluid temperature response

$$G_2 = \exp(-Ntu)L^{-1}\left\{\exp\left(\frac{Ntu}{s+1}\right)\hat{g}(s)\right\}$$

$$= \exp(-Ntu)\int_0^\tau \left\{\delta(\sigma) + e^{-\sigma} \times \frac{Ntu I_1(2\sqrt{Ntu\sigma})}{\sqrt{Ntu\sigma}}\right\} G_1(\tau - \sigma)\,d\sigma$$

$$= \exp(-Ntu)\left\{\int_0^\tau G_1(\tau - \sigma)\delta(\sigma)\,d\sigma + \int_0^\tau G_1(\tau - \sigma)\frac{e^{-\sigma}Ntu I_1(2\sqrt{Ntu\sigma})}{\sqrt{Ntu\sigma}}\,d\sigma\right\}$$

$$= \exp(-Ntu)\left\{G_1(\tau) + \int_0^\tau G_1(\tau - \sigma)\,.\,R(\sigma)\,d\sigma\right\}$$

where

$$R(\sigma) = e^{-\sigma} \cdot \frac{Ntu I_1(2\sqrt{Ntu\sigma})}{\sqrt{Ntu\sigma}}$$

With non-dimensional inlet disturbances $D$ given in Table 2, the general solution for outlet fluid temperature response becomes

$$G^{\#} = \exp(-Ntu)\left\{D(\tau) + \int_0^\tau D(\tau - \sigma) \cdot R(\sigma)\, d\sigma\right\} \tag{8}$$

*Table 2*

| Non-dimensional inlet disturbance, $D(\tau)$ | | At $x = 0$ |
|---|---|---|
| Step | 1 | |
| Exponential | $1 - k \exp(-\tau/\tau^*)$ | $\tau/\tau^* = t/t^*$ |
| First harmonic | $a_0 + a_1 \cos \bar{\omega}\tau + b_1 \sin \bar{\omega}\tau$ | $\bar{\omega}\tau = \omega t$ |

When solid temperatures are required, combining eqs. (6) and (7)

$$\hat{B} = \hat{g}(s) \frac{1}{s+1} \exp\left\{\left(\frac{1}{s+1} - 1\right)\xi\right\} \tag{9}$$

*At outlet*

$$\xi = Ntu$$

$$\hat{B}_2(Ntu, s) = \hat{g}(s) \cdot \frac{1}{s+1} \exp\left\{\left(\frac{1}{s+1} - 1\right) Ntu\right\}$$

Applying inverse Laplace transforms to outlet matrix temperature response

$$B_2 = \exp(-Ntu) L^{-1}\left\{\frac{1}{s+1} \exp\left(\frac{Ntu}{s+1}\right) \hat{g}(s)\right\}$$

$$= \exp(-Ntu) \int_0^\tau e^{-\sigma} I_0(2\sqrt{Ntu\sigma}) \cdot G_1(\tau - \sigma)\, d\sigma$$

$$= \exp(-Ntu) \int_0^\tau G_1(\tau - \sigma) \cdot P(\sigma) \cdot d\sigma$$

where

$$P(\sigma) = e^{-\sigma} I_0(2\sqrt{Ntu\sigma})$$

With non-dimensional inlet disturbances $D$ given in Table 2, the general solution for outlet matrix temperature response becomes

$$B^{\#} = \exp(-Ntu) \int_0^\tau D(\tau - \sigma) \cdot P(\sigma) \cdot d\sigma \tag{10}$$

Temperatures elsewhere in the regenerator may be found by inserting other values for $\xi$ in eqs. (8) and (10) or by using fictitious values for $L$.

For step inlet disturbance it is easily shown that the temperature difference (gas–solid) at outlet is

$$(G^{\#} - B^{\#})_{\text{step}} = \exp(-Ntu\tau) I_0(2\sqrt{Ntu\tau})$$

that the slope of the outlet response at any point is

$$\frac{dG^{\#}}{d\tau} = \exp(-Ntu) \cdot R(\tau)$$

and that in terms of an independent parameter $a$, locus of maximum slope $NtuP(\tau) = (1 + \tau)R(\tau)$ is given by

$$\tau = \frac{a}{2} \frac{I_0(a)}{I_1(a)} - 1$$

subject to $2 \leqslant Ntu < \infty$ with $Ntu = (a^2/4\tau)$ and $\beta = (\tau/Ntu)$. This last relationship was obtained in more complicated form by Kohlmayr **(14)**, both expressions giving the identical curve shown shaded in Fig. 1.

Attempt to obtain a similar expression for locus of maximum slope for exponential inlet disturbance leads to the condition

$$\int_0^\tau \exp\left(\frac{\sigma}{\tau^*}\right) \cdot R(\sigma) \cdot d\sigma = \frac{\tau^* R(\tau)}{\exp(-\tau/\tau^*)}$$

Evaluation of this expression was not carried out. Instead the position of maximum slope was determined numerically during evaluation of $G^{\#}_{\text{exp}}$ response curves, and the results are plotted in Fig. 1.

## 4 RELATIVE ACCURACY OF METHODS OF EMPLOYING MATHEMATICAL OUTLET RESPONSE CURVES IN EVALUATION OF EXPERIMENTAL RESULTS

The $Ntu$ value corresponding to a given experimental outlet response curve is determined through seeking the mathematical outlet response prediction which has the identical shape. Four techniques of comparison have been proposed, namely

(1) Complete curve matching
(2) Maximum slope
(3) Initial rise
(4) Phase angle and amplitude

The first three are appropriate to single-blow methods of testing. 'Complete curve matching' may be by least squares fit or by using a direct optimization simplex method **(33)–(35)** for both step and exponential input disturbances. 'Maximum slope' has been used by Locke **(22)** and later by Howard **(25)** with step inputs for the case of varying longitudinal conduction in the matrix. For exponential input with zero longitudinal conduction, new $Ntu$ versus locus of maximum slope curves are presented in Fig. 1. Practicable fast response heaters have exponential time constants around $\tau^* = 0{\cdot}2$ giving

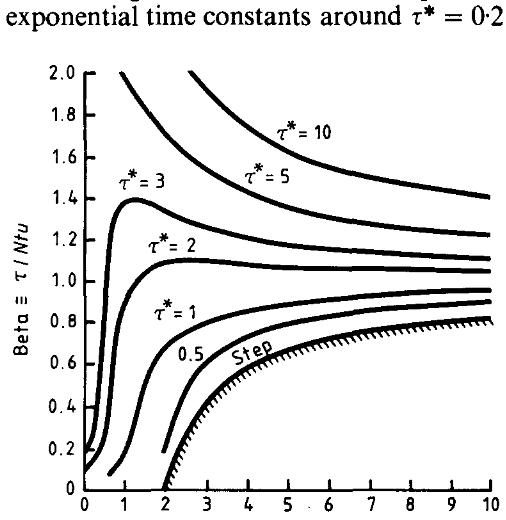


Fig. 1. Locus of maximum slope for experimental input

a locus of maximum slope curve close to that for step response. By choosing an inlet disturbance time constant $\tau^* = 2{\cdot}0$ an almost linear relationship between $Ntu$ and $\tau_{\text{max slope}}$ may be obtained, and additionally it becomes possible to evaluate experimentally, with some resolution, values of $Ntu$ down to about 1.

Figure 2 illustrates both step and exponential (dimensionless time constant $\tau^* = 0{\cdot}2$) response curves for zero longitudinal conduction calculated using eq. (8). The 'initial rise' technique proposed by Mondt and Siegla **(32)** makes use of the fact that the intercept of the response to step input at $\tau = 0$ has the value $\exp(-Ntu)$ from analytical solutions for both zero and infinite **(10)** longitudinal conduction in the matrix, it being postulated that the same result will hold for intermediate values. No heater has been devised which will produce a perfect step input **(36)**, and although output response curves for step and exponential input ($\tau^* = 0{\cdot}2$) are virtually identical down to $Ntu$ values of around 5 it is clear that the 'initial rise' method should be avoided completely, the 'maximum slope' method used only with knowledge of $\tau^*$, and that 'complete curve matching' is safest.

In present computations a top limit of $Ntu$ around 75 was the maximum attained before machine overflow occurred within the programme. Curves for values of $Ntu$ up to 500 have been obtained for the step test by Furnas **(23)** using graphical methods.

Figure 3 for first harmonic responses with ($a_0 = 1$, $a_1 = -1$, $b_1 = 0$, and $\bar{\omega} = 1{\cdot}0$) illustrates initial stages of steady-cyclic methods of testing, in which values of $Ntu$ may be calculated either from measurements of the ratio of amplitude of the varying fluid temperature at outlet to that at inlet, or alternatively from measurement of the phase lag between inlet and outlet fluid temperature variations. A separate theoretical analysis may be used when steady-cyclic conditions have been attained, e.g., Bel and Katz **(7)**, Meek **(8)**, and Shearer **(31)** who considered finite radial conductivity within the solid, and Stang and Bush **(24)** who examined the case of longitudinal conduction within the matrix.

On precision of the cyclic method Meek **(20)** observed some apparent variation in measured heat transfer $\tau^*$ alues against frequency, which he attributed to inaccuracies associated with very small downstream temperature amplitudes. Bell and Katz **(7)** advise 10 heating cycles before measurement of amplitude and phase angle are taken. For a given $Ntu$, Stang and Bush **(24)** showed that one frequency exists which will produce best test results for a given uncertainty in temperature measurement, recommending the cyclic method for values of $Ntu$ for the difficult range $0{\cdot}2 < Ntu < 5{\cdot}0$. However, from Fig. 2 there would seem to be less difficulty in resolving exponential response curves with complete curve matching using the single-blow technique.

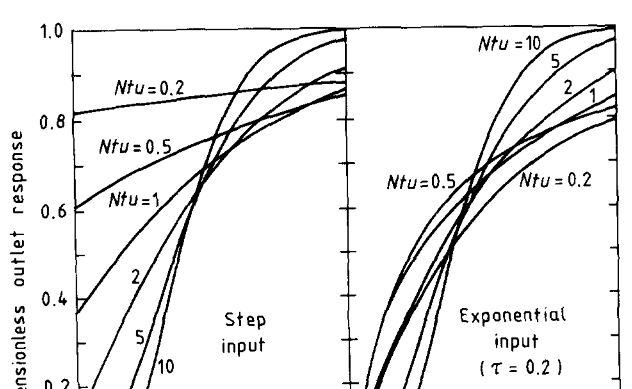


Fig. 2. Comparison of responses from step and experimental input disturbances

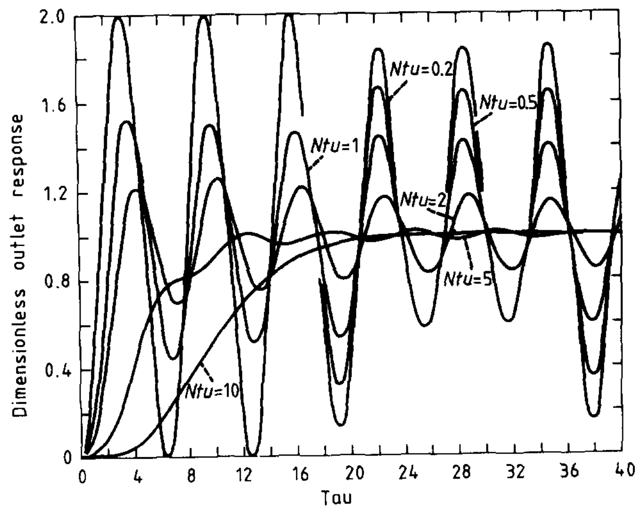


Fig. 3. First harmonic input $a_0 + a_1 \cos \hat{\omega}\tau + b_1 \sin \hat{\omega}\tau$ with $a_0 = 1$, $a_1 = -1$, $b_1 = 0$, $\hat{\omega} = 1{\cdot}0$

Finally, in obtaining theoretical response curves, two methods are available to the investigator

(5) Mathematical derivation of integral expressions which are subsequently evaluated numerically
(6) Direct application of numerical procedure to the physical problem

Kohlmayr **(17)** advises comparison of results from at least two, preferably several, independent curve matching methods. A general solution for method (5) has been presented above, to which comparison techniques (1)–(4) may be applied. The direct method (6) favoured by Dusinberre (discussion to **(37)**) has been employed, e.g., by Johnson **(38)**, Howard **(25)**, and other workers.

Additional effects which would complicate the canonical solution presented in this paper include

(1) Longitudinal conduction in the solid **(24)**, **(25)**
(2) Axial and longitudinal diffusion in the fluid **(18)**
(3) Surface losses from matrix exterior **(19)**, **(39)**
(4) Internal heat generation **(30)**, **(40)**, **(41)**
(5) Radial conduction in the solid interior **(5)**, **(20)**

## 5 CONCLUSIONS

(1) The 'initial rise' method for determining $Ntu$ is invalid for any practicable heater.
(2) The 'maximum slope' method requires accurate knowledge of heater exponential response time constant $\tau^*$, and should be used with caution when determining values of $Ntu < 5$.
(3) The 'maximum slope' method may give best resolution when $\tau^* = 2{\cdot}0$ for values of $Ntu$ down to 1.
(4) Practical complications exist with the 'periodic'

method in that Fourier analysis is required in order to extract the first harmonic from input outlet temperature waves.

(5) Single-blow testing with 'complete curve matching' by computer is recommended.


## 6 ACKNOWLEDGEMENT

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## 7 APPENDIX

Inverse Laplace transforms

$$L^{-1}\left\{\exp\frac{n}{s-a}\right\} = \delta(t) + e^{at}\frac{nI_1(2\sqrt{nt})}{\sqrt{nt}}$$

$$L^{-1}\left\{\frac{1}{s-a}\exp\frac{n}{s-a}\right\} = e^{at}I_0(2\sqrt{nt})$$

may be obtained by series expansion and term by term inversion.

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